NETWORK MODELLING OF INTERACTING CAPACITIVE IRISES
AND STEPS IN WAVEGUIDE

T. E. Rozzi
Philips Research Laboratories
Eindhoven, The Netherlands

Abstract

A unified approach is presented for modelling capacitive irises and steps in standard and oversize guides.

The variational solution yields an accurate wideband equivalent network with explicit, geometry-dependent element values. Design information is provided.

1. Introduction

Approximate monomode representations of discontinuities are familiar to the microwave engineer. No models at all, however, are available for configurations where more than one mode is propagating or still causes appreciable interaction. Recently a new approach to the modelling of transverse aperture-type discontinuities in a general multimode situation has been developed.[1] This approach is based upon the following physical observations:

i. Only the first few modes excited by a discontinuity "see" the successive discontinuity, that is, all the propagating modes plus possibly the first few nonpropagating ones. These we shall call accessible modes.
ii. All the infinite remaining modes can be considered as localized to the neighbourhood of the discontinuity which excites them.

Accessible modes represent the truly distributed part of the problem. Localized modes, instead, being almost "lumped" in nature, are responsible for the energy storage of the discontinuity. The total effect of the localized modes on the accessible ones can be represented by lossless, reciprocal, quasilumped multiports (one pair of accessible ports for each accessible mode). These multiports can be represented by a Foster canonical representation. The problem consists in extracting poles and residues directly from a Rayleigh-Ritz variational solution of the field problem.

The field problem of a cascade of discontinuities then reduces to the network problem of analyzing a cascade of lumped multiports connected by a finite (and usually small) number of transmission lines (see Fig. 1). The multiports are described by means of their canonical Foster form. The propagation constants of the transmission lines, one for each accessible mode, are known. Therefore, one can apply standard network analysis to obtain the overall characteristics of the cascade.

Inductive iris and step discontinuities under arbitrary TE(LSM) excitation have been treated in a recent paper.[2]

The same technique is also applicable to the treatment of obstacle problems under TM(LSE) excitation.

The present contribution treats the case of TM(LSE) excitation of aperture-type discontinuities or TE(LSM) excitation of obstacle-type discontinuities. The capacitive iris and step discontinuities are treated in detail.

2. Variational Solution

The basic geometry is illustrated in Fig. 2. If $b' = d$, the step is recovered; if $b' = b$ and $t = 0$, the infinitely thin iris is obtained. The k modes are accessible in guide g, k' modes in guide g' ($k + k' = k_a$). Let us introduce the transformation[2]

$$\cos \frac{\pi y}{b} \equiv P_{10} + P_{11} \cos \theta ;$$

$$P_{10} = \cos^2 \frac{\pi d}{2b} ;$$

$$P_{11} = 1 - P_{10} ; \qquad (1)$$

with the above transformation

$$\cos \frac{n\pi y}{b} = \sum_{p=0}^{n} P_{np} \cos p\theta ;$$

$$\cos \frac{m\pi}{b'} y = \sum_{p=0}^{\infty} A_{mp} \cos p\theta ; \qquad (2)$$

and the normalized susceptive part of the Green's function, i.e., the contribution of the localized modes only, reduces to the matrix

$$B_{pq} = \sum_{n>k} \frac{\beta}{\Gamma_n} P_{np} P_{nq} \varepsilon_{p0} \varepsilon_{q0} + \sum_{m>k'} \frac{\beta}{\gamma_m} A_{mp} A_{mq} \varepsilon_{p0} \varepsilon_{q0} \qquad (3)$$

$$\varepsilon_{p0} = \varepsilon_{0p} = 2 \text{ if } p = 0, = 1 \text{ if } p > 0 .$$

The propagation constants are:

$$\Gamma_n = \left[\left(\frac{\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \left(\frac{2\pi}{\lambda}\right)^2\right]^{1/2} ;$$

$$\gamma_m = \left[\left(\frac{\pi}{a}\right)^2 + \left(\frac{m\pi}{b'}\right)^2 - \left(\frac{2\pi}{\gamma}\right)^2\right]^{1/2} ; \qquad (4)$$

$$\Gamma_0 = j\beta = \gamma_0 .$$

We define also:

$$\bar{\omega} = \frac{2a}{\lambda} ;$$

$$\beta = \frac{a}{\pi} \bar{\beta} = \sqrt{\bar{\omega}^2 - 1} , \; \delta_n = \frac{b}{na} .$$

The Rayleigh-Ritz variational expression of order N for the reactance matrix of the iris, as seen by the accessible modes, is

$$x = x^T = Q \cdot B^{-1} \cdot Q^T \quad (5)$$

where Q denotes the matrix formed with the first k rows of P and k' rows of A and the infinite matrix B has been replaced by its N x N truncation. Since $Q_{k+1,*} = \sqrt{\frac{b}{b'}} Q_{1,*}$, we need consider only $k_a - 1 = \bar{k}$ ports.

3. Frequency Dependence

Localized modes are "quasilumped" in nature. Therefore an effective positive real (p.r.) approximation of the modal characteristic admittance is obtained by means of a continued fraction expansion of the square root in (4).

Successive truncations yield:

$$\beta/\Gamma \simeq \bar{\beta}\delta_n , \; \frac{\delta_n \bar{\beta}}{1 - \frac{1}{2} (\delta_n \bar{\beta})^2} , \; \frac{1}{3} \bar{\beta}\delta_n + \frac{\frac{2}{3} \bar{\beta}\delta_n}{1 - \frac{3}{4} (\delta_n \bar{\beta})^2} . \quad (6)$$

Let us use in (3) the third-order approximation for $n \le n_d$ (n_d arbitrary) and the first-order (quasistatic) approximation for $n > n_d$. A similar approximation is valid "a fortiori" in guide 2.

Setting $\bar{k} = N$, (5) yields

$$x^{-1} = \left(Q \cdot B^{-1} \cdot Q^T\right)^{-1} \equiv U^T \cdot B \cdot U \equiv y . \quad (7)$$

Introducing (6) in the above equation, we obtain a canonical Foster representation of the iris susceptance y as seen by $\bar{k} = N$ accessible modes

$$y(\bar{\beta}) = C^S \bar{\beta} + \sum_{n=k}^{nd} \frac{\bar{\beta} r^{(n)}}{1 - \left(\frac{\bar{\beta}}{\bar{\omega}_m}\right)^2} + \sum_{m=k^1}^{md} \frac{\bar{\beta} r'^{(m)}}{1 - \left(\frac{\bar{\beta}}{\bar{\omega}'_m}\right)^2} \quad (8)$$

where m_d is the equivalent of n_d in guide 2 and

$$S = \eta - \sum_{n=1}^{nd} D^{(n)} + \sum_{m>m_d} D'^{(m)} + \sum_{n=k}^{nd} \frac{1}{3} D^{(n)} + \sum_{m=k'}^{md} \frac{1}{3} D'^{(m)}$$

$$D^{(n)}_{pq} = \delta_n P_{np} P_{nq} \varepsilon_{p0} \varepsilon_{q0} \; ; \; D'^{(m)}_{pq} = \delta_m A_{mp} A_{mq} \varepsilon_{p0} \varepsilon_{q0}$$

$$\eta_{pq} = \frac{b}{a} \begin{cases} 0 & : \; p \ne q \\ 4\ell n \; \csc \frac{\pi d}{2b} & : \; p = q = 0 \\ 1/p & : \; p = q > 0 \end{cases}$$

$$C^S = U^T \cdot S \cdot U \; ; \; r^{(n)} = \frac{2}{3} U^T \cdot D^{(n)} \cdot U \; ;$$
$$r'^{(m)} = \frac{2}{3} U^T \cdot D'^{(m)} \cdot U^T \; ;$$

$$\bar{\omega}_n = \frac{2}{\sqrt{3}} \frac{1}{\delta_n} \; ; \; \bar{\omega}' = \frac{2}{\sqrt{3}} \frac{1}{\delta'_m} \; ; \; \delta'_m = \frac{d}{am} .$$

If $\bar{k} < N$ the unwanted $N - \bar{k}$ ports are left open. It appears from (3) that the case $\bar{k} > N$ is not interesting.

In the case of a thick iris in a uniform guide, it is possible to make use of longitudinal symmetry. Canonical representations have been derived in this case too.

4. Examples

Figure 3 depicts geometry, equivalent circuit and element values versus b/d of the E-plane step obtained by setting $N = n_d = m_d = \bar{k} = 1$ in (8). The reflection coefficient is accurate within 2 percent over the band $1 < \bar{\omega} < 2$ over the whole step range.

Figure 4 shows the equivalent network of an infinitely thin iris with two accessible modes. This was obtained by setting $\bar{N} = k = n_d = 2$ and is applicable to the situation of an iris in an oversized guide or, even in a standard guide, when more discontinuities are placed close to each other. The elements of the equivalent network, as functions of b/d, have been derived in close form.

Figure 5 displays the transmission coefficient of two identical, symmetrical, infinitely thin irises spaced $\lambda g/8$ apart at one spot frequency for a varying iris aperture. The experimental points and curve a have been derived from [3]. Curve b has been computed with the equivalent circuit of Fig. 4. Introducing the effect of a small iris thickness (curve c) improves the agreement still further.

Conclusions

True wideband equivalent circuits of capacitive irises and steps have been derived from a Rayleigh-Ritz variational solution. The element values are functions of the geometry only. These results can be used directly in a network synthesis program.

Acknowledgement

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References

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3. H. Bosma, "Two capacitive windows in rectangular waveguide," Appl. Sci. Res., vol. B7, pp. 131-144, 1958.

Cascade of interacting discontinuities

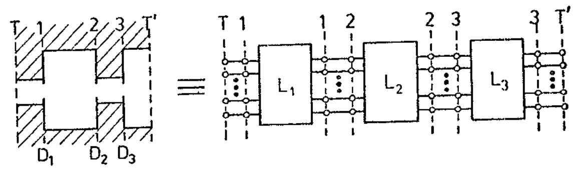
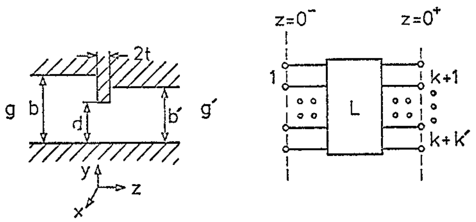


Figure 1



a) Iris/step geometry b) network representation

Figure 2

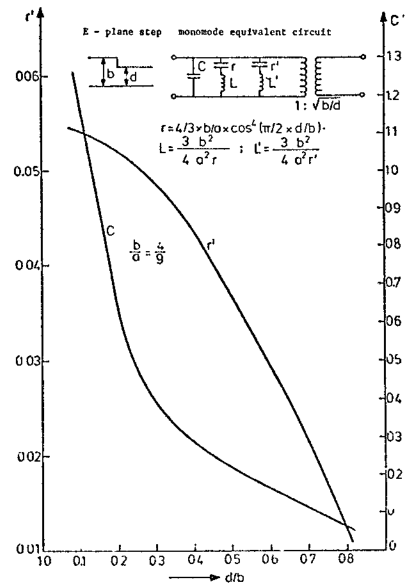
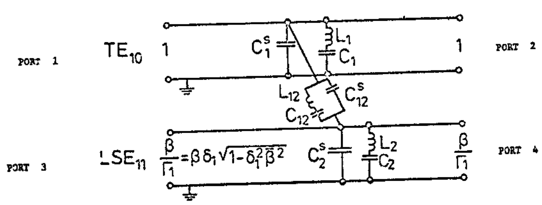


Figure 3



Infinitely thin iris, two accessible modes (N = k = n_d = 2). Element values in appendix

Figure 4

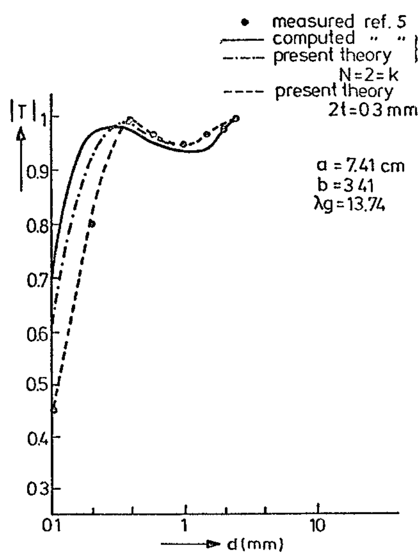


Figure 5

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